Comments on "Carrier Tracking in RAKE Reception of Wide-Band DSSS in Ricean Fading"

R. E. Ziemer and L. B. Milstein

***Abstract*—In an earlier paper by Ziemer *et al.*, the effect of phase-tracking error in the fingers of a coherent RAKE receiver was characterized. This paper gives additional results for the case where the phase tracking error is due jointly to input noise to the receiver and phase noise internal to the phase tracking device. An optimum number of paths exists, giving a minimum bit-error probability.**

***Index Terms*—Direct-sequence spread spectrum, RAKE reception, Riccan fading.**

In the above paper,[1] a technique for characterizing the effects of phase-tracking error in the fingers of a coherent RAKE receiver was given (see the above paper and [1]). The performance curves shown in Fig. 1 incorporate the corrections noted in [1]. In the above paper, the conditions for the noise variance in the phase-tracking loops were not stated clearly—the curves shown were for constant signal-to-noise ratio (SNR) in each finger with each finger SNR weighted by the power delay profile (PDP). Note that the curves shown in Fig. 1 are similar to those of Fig. 1 in the above paper, except that the $(1+K)^{-1}$ factor inadvertently inserted in the code in the above paper, as noted in [1], made the apparent loop SNRs lower than they actually were (consequently, the curves in Fig. 1 of this paper are slightly lower than those of Fig. 1 of the above paper).

In this paper, we generalize the results of the above paper somewhat and let the SNR in the kth finger of the RAKE receiver, i.e., $\mathrm{SNR}(k)$, be

$$\mathrm{SNR}(k) = \frac{E_b R_b}{N_0 B_L + \sigma_{\mathrm{int}}^2} P_{\mathrm{pdp}}(k) \tag{1}$$

where E_b is the bit energy, R_b is the data rate, N_0 is the additive white Gaussian noise (AWGN) spectral density at the receiver input, B_L is the equivalent noise bandwidth of the phase tracking loops, σ_{int}^2 is the mean-square value of the noise internal to the phase tracking loops, and $P_{\mathrm{pdp}}(k)$ gives the distribution of received signal power in the fingers of the RAKE receiver (i.e., the PDP). We rewrite (1) as

$$\mathrm{SNR}(k) = \frac{E_b}{N_0} \frac{R_b}{B_L} \frac{P_{\mathrm{pdp}}(k)}{1 + \dfrac{\sigma_{\mathrm{int}}^2}{N_0 B_L}}. \tag{2}$$

Note that (2) is conveniently grouped into terms that are all dimensionless.

The internal noise variance in (2) can be shown to be proportional to B_L^{-n}, where B_L is the loop bandwidth and the exponent value is dependent on the type of phase noise of the controlled oscillator within the phase tracking device (e.g., $n = 1$ for random phase walk [2, Appendix D]). Therefore, an optimum B_L exists that provides a minimum in the phase-error variance due to both additive input noise and tracking oscillator jitter [3]. When the optimum B_L is used, the ratio of internal to external noise variances in (2) is $\sigma_{\mathrm{int}}^2/N_0 B_L \approx 1$.

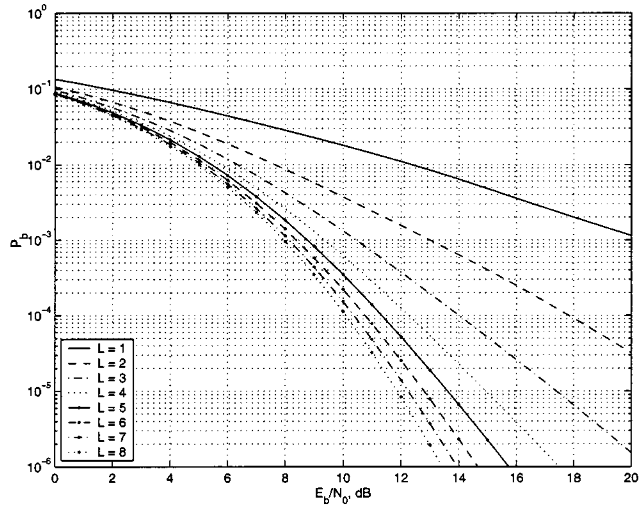

Fig. 1. P_b versus E_b/N_0 in Ricean fading with $K = 0$ dB and constant PDP. Loop SNR 20 dB above $E_b/N_0 = 0$ dB with power equally divided between fingers. L is the number of RAKE fingers.

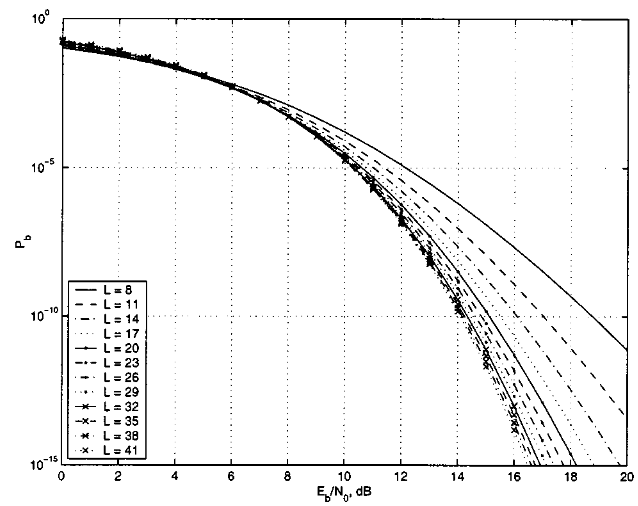

Fig. 2. P_b versus E_b/N_0 for various orders of diversity, L, in Ricean fading: $K = 6$ dB, $\sigma_{\mathrm{int}}^2/N_0 B_L = 1$, $R_b/B_L = 15$ dB, exponential PDP.

Using the computational procedure of the above paper, typical results may be generated, as shown in Figs. 2–6, for various parameter values. For example, Fig. 2 shows bit-error probability P_b versus E_b/N_0 for a Ricean fading channel with a specular-to-diffuse fading power ratio of $K = 6$ dB and the ratio $\sigma_{\mathrm{int}}^2/N_0 B_L = 1$ for orders of diversity $L = 8 - 41$ in steps of three with the PDP $P_{\mathrm{pdp}}(k)$ in (2) exponentially decaying (smallest power component 20 dB below the largest component). A crossover of the curves exists showing that there is an optimum value of diversity. These optimum values are illustrated by plotting P_b versus L for an exponential PDP in Fig. 3 for $K = -6,\ 0,$ and 6 dB, $\sigma_{\mathrm{int}}^2/N_0 B_L = 1$, $E_b/N_0 = 7$ dB, and $R_b/B_L = 15$ dB. Optimum L-values are 37, 34, and 26, respectively, for $K = -6,\ 0,$ and 6 dB.

The effect of varying $\sigma_{\mathrm{int}}^2/N_0 B_L$ is illustrated by Fig. 4, which shows P_b versus L for $\sigma_{\mathrm{int}}^2/N_0 B_L = 0,\ 1,$ and 2 in a Ricean fading channel with $K = 6$ dB. All other parameters are the same as for

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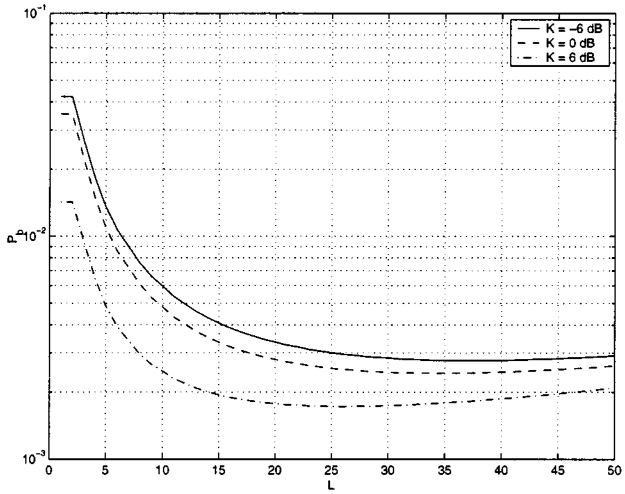

Fig. 3. P_b versus L. Ricean fading with $K = -6,\ 0,\ 6$ dB, $E_b/N0 = 7$ dB, $\sigma_{\rm int}^2/N_0B_L = 1$, $R_b/B_L = 15$ dB, exponential PDP. Optimum L values: 37, 34, and 26.

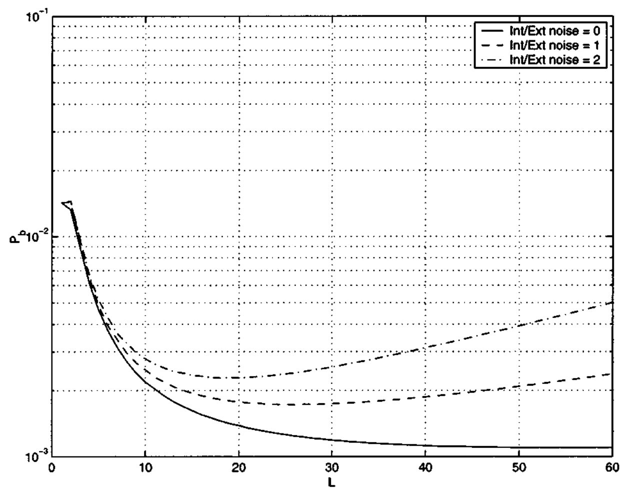

Fig. 4. P_b versus L. Ricean fading with $K = 6$ dB, $E_b/N_0 = 7$ dB, $\sigma_{\rm int}^2/N_0B_L = 0,\ 1$ and 2, $R_b/B_L = 15$ dB, exponential PDP. Optimum L values: 56, 26, and 20 for $\sigma_{\rm int}^2/N_0B_L = 0,\ 1,$ and 2, respectively.

Fig. 3. Optimum L-values are 56, 26, and 20 for $\sigma_{\rm int}^2/N_0B_L = 0,\ 1,$ and 2, respectively. Note, however, that little is gained in performance by going beyond $L = 15$ in any case.

Figs. 5 and 6 show sensitivity to E_b/N_0 and R_b/B_L, respectively, where it is seen that the optimum L increases with increasing E_b/N_0 and with increasing R_b/B_L.

As in the above paper, we conclude that finer multipath resolution, through wider spread bandwidth, buys improved performance up to the point where the degradation in one's ability to make accurate channel estimates outweighs the enhancement one would obtain due to increased diversity. However, while this improvement is very dramatic for a few RAKE fingers combined (say, five or so), it is less dramatic as the number of fingers goes beyond 15 or 20.

References

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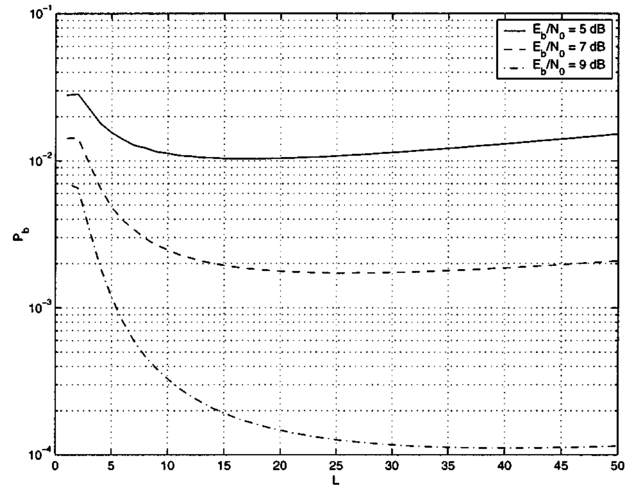

Fig. 5. P_b versus L. Ricean fading with $K = 6$ dB, $E_b/N_0 = 5,\ 7,$ and 9 dB, $\sigma_{\rm int}^2/N_0B_L = 1$, $R_b/B_L = 15$ dB, exponential PDP. Optimum L values: 18, 26, and 41 for $E_b/N_0 = 5,\ 7,$ and 9 dB, respectively.

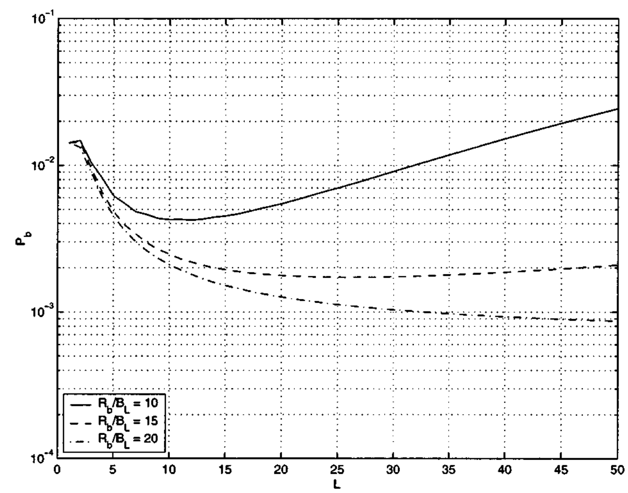

Fig. 6. P_b versus L. Ricean fading with $K = 6$ dB, $E_b/N_0 = 7$ dB, $\sigma_{\rm int}^2/N_0B_L = 1$, $R_b/B_L = 10,\ 15,$ and 20 dB, exponential PDP. Optimum L values: 12 and 26 for $R_b/B_L = 10$ and 15 dB, respectively.

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